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ANCHOR MOORING LINE COMPUTER PROGRAM USER'S MANUAL FOR CHAIN-SOIL ANALYSIS PROGRAM (CSAP)

by

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This FORTRAN computer mooring lines associated seafloor. Any combination line. The program calculator surface buoy. The program addition, either the total leanchor and the buoy or the	with drag/fixed anchor on of chain and cable sates a detailed solution am requires horizontal ength of the mooring l	rs embedded in a cohesive segments may be used to a of the mooring line from tension at the buoy as paine or the horizontal distant.	e or cohesionless specify the mooring the anchor to the rt of the input. In ance between the	
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ANCHOR MOORING LINE COMPUTER PROGRAM (Phase II)

Final Report

User's Manual for Program CSAP2 (Version 2)

Submitted to

Naval Facilities Engineering Service Center

by

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PROGRAM ABSTRACT

TITLE:

Chain - Soil Analysis Program (CSAP2) - Version 2

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DESCRIPTION:

This FORTRAN computer program performs the design and analysis of single-leg chain/cable mooring lines associated with drag/fixed anchors embedded in a cohesive or cohesionless seafloor. Any combination of chain and cable segments may be used to specify the mooring line. The program calculates a detailed solution of the mooring line from the anchor to the surface buoy. The program requires horizontal tension at the buoy as part of the input. In addition, either the total length of the mooring line or the horizontal distance between the anchor and the buoy or the angle of the mooring line at the seafloor surface can be specified.

HIGHLIGHTS OF VERSION 2:

The major improvements included in version 2 of the CSAP program are inclusion of (1) cohesionless seafloor soil and (2) force - deformation (P-DELTA) relationship at the buoy. In addition, the elastic stretch of the entire mooring line at the time of deployment is also calculated. Other minor technical improvements have also been made to enhance the analytical performance of the program. The details are explained in this user's manual. Please note that data files created by the previous version of the CSAP program can not be read by this new version because of the additions made in input.

FORMULATION:

The analysis of mooring lines at deployment is based on the limit equilibrium approach in which the detailed solutions are determined from the equilibrium condition. The solutions are obtained from recursion formulas that start at the seafloor surface and propagate toward both the buoy and the anchor.

The force - deformation (P-DELTA) relationship at the buoy describes the postdeployment behavior of the mooring line under various loads applied at the buoy. It is based on the assumption that all materials behave completely elastically.

PROGRAM STRUCTURE:

The program is written in FORTRAN 5 for IBM compatible personal computers. The DOS version of the program (CSAP2.EXE) is compiled in such a way that the program can be run with or without a math coprocessor. The WINDOWS version of the program (CSAPW2.EXE) can be run within the WINDOWS environment.

The core storage requirements are approximately 125 K and 156 K bytes, respectively, for DOS and WINDOWS versions of the executable program. Input is from either the keyboard or a data file. Output is written to a file whose name is specified during the input. In addition, a separate output file in ASCII format is generated in case other software is used for graphically plotting the results. The DOS version of the program includes its own graphics subroutine from which general two-dimensional plots between any pair of parameters may be viewed. The WINDOWS version, however, does not have its own graphics capability because of a conflict with the compiler.

RESTRICTIONS:

The program converts relatively simple input into necessary parameters for the detailed analysis. It therefore includes certain restrictions. If any of the restrictions are violated, the program displays a description of the reason and in most cases asks for new

input. If FORTRAN 5 compiler language rules are violated in input, it displays error message(s) that are provided by the compiler (MICROSOFT FORTRAN COMPILER version 5.1) with simple explanations.

The program has the following restrictions: Number of elements of 3010; Number of mooring line segments of 10; Number of sinkers of 10; Number of soil strength data points of 20.

INTRODUCTION

This computer program performs the design and analysis of single-leg chain/cable mooring lines associated with drag/fixed anchors embedded in a cohesive or cohesionless seafloor. Any combination of chain and cable segments may be used to specify the mooring line. The program calculates a detailed solution of the mooring line from the anchor to the surface buoy. It is based on the assumption that all materials behave completely elastically. The program requires horizontal tension at the buoy as part of the input. Either the total length of the mooring line or the horizontal distance between the anchor and the buoy or the angle of the mooring line at the seafloor surface can be specified. The program also calculate the force - deformation (P-DELTA) relationship at the buoy to describe the post-deployment behavior of the mooring line under various loads applied at the buoy.

The program is capable of analyzing mooring lines embedded in either cohesive or cohesionless soils, or soils having both cohesion and friction. However, experimental verification has only been made through centrifuge model tests on soft clays. Use of the program with other soils than soft clays therefore should be done with caution.

This manual is divided into five sections. The first section includes a brief description of the input parameters. The second section contains a detailed explanation of the input parameters. The third section describes the output of the program. The fourth section illustrates example problems and the fifth section contains the program details including the flow charts.

The program uses four files in addition to the keyboard input and the monitor output. File 6 contains the detailed solution output generated by the program. When a new set of input data is entered, file 8 is used to create a duplicate input data file which can be read by the program later for additional analysis. When the existing input data file (file 9) is called, the user can run the problem either with or without changes in data. In

either case, the program creates a new data file. In addition, file 7 is generated to store output data in ASCII format so that the user may use other software for graphical plotting. It is therefore important to record all input and output file names for future use.

Only the master file name is specified by the user. Names of the output file, the ASCII data file, and the duplicate input file will be created by the program with suffixes of 1, 2, and 3, respectively. Please do not include any blank space(s) when inputting the master file name because any character after the blank space is ignored. A maximum of seven characters may be used for the master file name.

The program can be run under either the DOS or the WINDOWS environment. To run the DOS version of the program, you must be at DOS prompt and move to a directory or subdirectory where the program CSAP2.EXE resides. Typing and entering CSAP2 starts the program. All files described in above paragraph will be genetrated in the same directory/subdirectory.

The WINDOWS version of the program can be run in several ways.

- 1. From DOS, move to the directory where CSAP2W.EXE is located. Then type "WIN CSAP2W".
- From WINDOWS, double click the FILE MANAGER icon to activate the file manager. Move to and open the directory containing CSAP2W.EXE. Double click CSAP2W.EXE.
- 3. From WINDOWS, pull down the FILE menu and click RUN. Type in the complete path where CSAP2W.EXE is located, e.g., c:\.....\csap2w.exe.
- 4. From WINDOWS, create a file group and a PIF file to run CSAP2W.EXE.

When you execute the WINDOWS version of the program, you will see four "child windows." They have titles of UNIT *, ASCII DATA OUTPUT, CSAP OUTPUT, and CSAP INPUT. The first one will not be used. The second one is for the ASCII file which will generate five column data for graphical plots using other software. The third one is for the solution output. The last one is for communication between the

user and the computer, i.e., data input. The last one will be the main window to begin with. Within the CSAP INPUT window, input data as directed. As outputs are generated, you will see the outputs being written in the CSAP OUTPUT and ASCII DATA OUTPUT windows. You can see them better if you arrange the child windows in "tile" fashion rather than "cascade" fashion. The child windows can be scrolled up or down even after the program execution is terminated. This will let you review the previous screens of input and/or output.

CSAP2W.EXE will not generate the output file and the ASCII data file, since they are displayed in child windows. You can save them from the child windows using the EDIT, SELECT ALL, and COPY commands. However, the input data will be stored in a file name you designated (with a suffix "3") and can be recalled anytime.

SECTION I - INPUT

The program itself is virtually self-explanatory. As the calculations proceed, the program will continuously ask for input of information with detailed instructions to the user. Please make sure that correct units for input parameters are used.

Note that the program is written using mostly free format for data input, unless specified otherwise. When inputting parameters in free format, the data, if more than one, must be separated by blank space(s) or comma(s). Please enter all data starting in column 1.

Following is a complete list, in order, of the input parameters

1. TITLE: Heading of the output file (columns from 1 to 80).

2. INPT: 1 to input data from keyboard.

2 to use existing data file.

If INPT = 1, go to step 4.

3. FILEIN: Name of the data file.

4. FNAME: Master output file name (up to 7 characters).

If INPT = 2, data from existing file will be displayed on the screen in order. The user will be able to change any of the input parameters. If INPT = 1, follow the steps described below.

5. IS: Solution criterion.

1 for horizontal distance between the anchor and the buoy.

2 for total mooring line length between the anchor and the buoy.

3 for mooring line angle at the seafloor surface.

6. ERMAX: Maximum allowable error for convergence (Default = 0.005).

7. NC: Number of mooring line segments between the anchor and the buoy.

Must be between 1 and 10. Input the following for NC times. The

segment number starts from the anchor.

TY(I): Type of the Ith segment material.

C for chain and W for cable.

D(I): Diameter of the Ith segment material.

Enter nominal link diameter for chain or total diameter for cable.

W(I): Buoyant weight of the Ith segment material per linear foot.

Return uses the default value.

AE(I): Axial stiffness of the Ith segment material.

Return uses the default value.

XL(I): Length of the Ith segment material.

If NC = 1, go to step 9.

8. NCC: Number of sinkers. If NCC = 0, go to step 9. If not, input the

following for NCC times.

I: Segment number whose end the sinker is attached to.

WC(I): Buoyant weight of the sinker attached to the end of Ith segment.

9. HW: Depth of water.

TH: Horizontal tension applied at the buoy.

DS: Element length for the analysis. Default = 1 ft.

H: Depth from the seafloor surface to the anchor.

XG: From step 5, if

IS = 1, horizontal distance between the anchor and the buoy.

IS = 2, total mooring line length.

IS = 3, mooring line angle to the horizontal at the seafloor surface.

10. IET: Input data control. Return to use defaults for EWBC, EWBW,

EWSC, EWSW, XNCC, XNCW, ALPC, ALPW, BETAC, BETAW.

DELC, DELW.

EWBC: Equivalent diameter conversion factor of chain for normal force.

Default = 0.3

EWBW: Equivalent diameter conversion factor of cable for normal force.

Default = 0.0833

EWSC: Equivalent diameter conversion factor of chain for sliding force.

Default = 0.94

EWSW: Equivalent diameter conversion factor of cable for sliding force.

Default = 0.2618

XNCC: Bearing capacity factor for chain (Default = 10.0).

XNCW: Bearing capacity factor for cable (Default = 9.0).

ALPC: Soil adhesion conversion factor for sliding force for chain (Default = 1.0).

ALPW: Soil adhesion conversion factor for sliding force for cable (Default = 1.0).

BETAC: Contact area conversion factor for sliding force for chain (Default = 1.0).

BETAW: Contact area conversion factor for sliding force for cable (Default = 1.0).

DELC: Soil friction conversion factor for sliding force for chain (Default = 1.0).

DELW: Soil friction conversion factor for sliding force for cable (Default = 1.0).

11. NS: Number of points defining the soil strength variation with depth.

Must be between 1 and 20. Input the following for NS times. The

number starts from the seafloor surface and downward.

ZT(I), SUT(I), PH(I), GAM(I): Depth, cohesion, internal friction angle and unit weight of the seafloor soil at Ith point.

If IS = 3 in step 5, skip the step 12.

12. EIN: Initial angle of the mooring line to the horizontal at the seafloor surface for

solution search (Default = 0.1 deg).

For the WINDOWS version of the program, go to step 15. For the DOS version, the following inputs are needed for graphical plotting on screen.

13. IY: Y if plotting of the results is desired. N moves the user to step 15.

14. ICX: Abscissa of the plot

1 for X coordinate of mooring line.

2 for Y coordinate of mooring line.

3 for mooring line length coordinate.

ICX1: Plotting option

1 for seafloor portion of mooring line only 2 for catenary portion of mooring line only

3 for entire mooring line.

ICY: Ordinate of the plot

for X coordinate of mooring line.
 for Y coordinate of mooring line.
 for mooring line length coordinate.

4 for mooring line tension force.

5 for mooring line angle to the horizontal.

15. IAP: Y to calculate the P-DELTA relationship. N moves to step 16.

16. IAP: Y to solve another problem. N terminates the program.

SECTION II - EXPLANATORY INFORMATION

Described below is the explanatory information regarding the input contained in Section I. Note that the numbering is the same as Section I to facilitate the consultation of the explanatory information.

1. TITLE:

The data set for each problem begins with a title. There are no restrictions or requirements regarding the contents of the title. It can be up to 80 characters.

2. INPT:

The data may be inputted from the keyboard or a data file. In either case the inputted data will be written to a file whose name is specified by FILEIN.

3. FILEIN:

This is the file name from which the input data is to be read. The last character of FILEIN is numeric letter "3". See step 4 for further details.

4. FNAME:

This is the master output file name specified by the user. If this name already exists in the directory where the program resides, an error message will appear. You need to reenter a new name. It can be up to 7 alphanumeric characters.

Three sub-files will be created from FNAME. The complete output from the analysis will have a name specified by FNAME with a number "1" attached at its end. An output data file in ASCII format will also be created with a suffix "2". This file contains in its first line the total number of data points and in succeeding lines five columns of data that include the X coordinate, Y coordinate, length coordinate, angle to

the horizontal in degrees, and developed tensile force at the end of each element. Finally, the inputted data will be copied to a file with a suffix of "3".

In designating the name of FNAME, do not include any blank space(s) because any character after a blank will be ignored. A maximum of 7 characters may be used for FNAME.

5. IS:

The program searches for the solution based on the criterion specified by the user. The solution criterion can be one of the following three: (1) horizontal distance between the anchor and the buoy (IS = 1), (2) total mooring line length between the anchor and the buoy (IS = 2), or (3) mooring line angle to the horizontal at the seafloor surface (IS = 3).

6. ERMAX:

The solution is obtained from an iterative search. If the relative error between any two successive iterative values specified by IS (step 5) is less than ERMAX, the program assumes that the convergence has been obtained and therefore terminates the analysis.

7. NC, TY(I), D(I), W(I), AE(I), XL(I)

Any combination of chain or cable segments, up to 10, may be used to describe the mooring line from the anchor to the buoy. A segment is defined as a continuous length of the same type of material (chain or cable) with the same diameter. Sinkers must be attached at the end of the segment. Therefore, if a sinker exists in the middle of a same diameter chain or cable, it needs to be divided into two segments with identical material properties and diameters.

The type of segment, TY(I), can be either chain or cable. In the former case, enter either upper or lower case "C". For a cable segment, enter either upper or lower case "W".

1

The default weights of the mooring line are calculated assuming the properties of steel as shown below. If synthetic wires are used, the user needs to enter the correct buoyant weights of the cables per foot, i.e., in lbs/ft.

$$W = 8.628696 \times D^2$$
 for chains

$$W = 1.61 \times D^2$$
 for cables

where D is in inches and W is in lbs/ft.

The axial stiffness of the mooring line, AE, is used to calculate the total elastic stretch of the mooring line at the time of deployment as well as the P-DELTA relationship. The default axial stiffness of the mooring line segments are estimated assuming the properties of steel as shown below. If synthetic wires are used, the user needs to enter the correct axial stiffness of the cables in lbs.

$$AE = 8.595 \times D^2$$
 in million lbs for forged steel stud link chains $AE = 5.517 \times D^2$ in million lbs for $6x19$ wire ropes where D is in inches.

The length of the last segment, i.e., the one attached to the buoy, will be set to an arbitrarily large value regardless of the user's input. This is necessary because during the solution search a very large length of the mooring line may be needed.

8. NCC, WC(I):

All sinkers must be attached to the end of the mooring line segments. The weight of the sinker, WC(I), must be the buoyant weight. Buoys can be described by specifying their buoyancy with a corresponding negative weight.

9. DS, XG:

The entire mooring line will be divided into elements whose length is specified by DS. Since "segment" defines the material properties, the length of the element must be smaller than the shortest segment length defining the mooring line.

The value of XG depends on the solution criterion specified in step 5.

10. EWBC, EWBW, EWSC, EWSW, XNCC, XNCW, ALPC, ALPW, BETAC, BETAW, DELC, DELW:

The equivalent diameter conversion factor of chain for normal force (EWBC) can be obtained directly from the conversion of chain geometry to the chain bearing area. If only the frontal area of the cylinder defined by a circle that encompasses the two perpendicular chain links is assumed for the chain bearing area, the equivalent bearing area (ft²/ft) becomes 0.3 times the chain nominal size (link diameter in inches). For cables, the direct conversion of the diameter in inches to ft²/ft shows that the value of the equivalent diameter conversion factor for normal force (EWBW) is 0.0833.

The equivalent diameter conversion factor of chain for sliding force (EWSC) can also be obtained directly from the conversion of chain geometry to the chain surface contact area. The circumference of a cylinder defined by chain links is $(3.6 \times \pi \times D) / 12$ or $0.942 \times D$ ft²/ft. The default value of EWSC is therefore taken as 0.94. For cables, the direct conversion of the diameter in inches to ft²/ft shows that the value of the equivalent diameter factor of cable for sliding force (EWSW) is 0.2618.

The bearing capacity factor for chain (XNCC) was obtained from the results of fourteen field tests with several different types of anchors embedded in cohesive seafloor by the Naval Civil Engineering Laboratory (Ref. 1). They show that the bearing capacity factor for chain is 10.

The bearing capacity factor for cable (XNCW) is obtained from the bearing capacity of a deeply buried strip footing under an undrained condition, i.e., $q_{ult} = 9 \times S_u$ (Ref. 2). Therefore, the default value of the cable bearing capacity factor is taken as 9.0.

The soil adhesion conversion factors for sliding force (ALPC and ALPW) convert the soil cohesion into adhesion. For chains, the value of ALPC can be taken as 1.0 due to the nature of chain links formation. For cables, the value of ALPW can be estimated from the behavior of piles. It starts at 1.0 for soil cohesion up to 1,000 psf, linearly varies from 1.0 to 0.4 for cohesion between 1,000 and 6,000 psf, and remains constant at 0.4 for cohesion over 6,000 psf.

The contact area conversion factor for sliding force in cohesive soils (BETAC and BETAW) is the ratio between the true contact area and the total available contact area between the mooring line and the soil while sliding. The value is taken as 1.0 for normally consolidated clay soils. If the mooring line starts to separate from the soil on its back side, the values of BETA could approach 0.5.

The soil friction conversion factor for sliding force (DELC and DELW) is the ratio between the interface friction angle (between the soil and the mooring line) and the soil internal friction angle. The default is taken as 1.0.

The conversion factors for soil adhesion, contact area, and soil friction directly influence the results of the analysis. A comprehensive experimental study therefore needs to be conducted to provide data and establish guides for more accurate estimation of these factors.

11. NS, ZT(I), SUT(I), PH(I), GAM(I):

NS specifies the total number of data points describing the seafloor soil material properties variation with depth. If only one point is used, the soil is assumed to have a constant material properties profile. If two or more points are specified, the soil material

properties variation is obtained from interpolation between the data points and extrapolation beyond the boundary data points.

When the seafloor consists of soil layers with sudden changes in material properties, e.g., non-continuous cohesion, friction angle, and/or unit weight, it is necessary to introduce a very thin fictitious soil layer between the layers of non-continuous properties so that the material properties across the non-continuous boundary become continuous. This introduction of a fictitious soil layer allows the correct interpolation of soil properties between the non-continuous soil layers.

In any case, the soil cohesion is set to be greater than 15 psf to avoid the possible "sink" of the mooring line near the seafloor surface when the soil is truly cohesive, i.e., zero friction angle.

For clay soils, the soil friction angle, PH(I), can be set to be zero. The buoyant unit weight of the soil, GAM(I), does not influence the results of the analysis of clay soils. For sandy soils, however, an accurate value of the soil buoyant unit weight must be entered. Typical values of the buoyant unit weight and friction angle of sands are 55 pcf and 34 degrees, respectively.

12. EIN:

The initial angle of mooring line to the horizontal at the seafloor surface may be specified to expedite the solution search process. However, it must be smaller than the true angle, since the search is limited to steeper angles. If the inputted value of EIN is greater than the true value, the program will ask the user to reenter the data.

13. IY:

Only the DOS version of the program has the built-in graphics subroutine that can display two dimensional plots. If the WINDOWS version of the program is used, the user can utilize any plotting software that reads data in ASCII format. The ASCII data

file described in step 4 can be used for this purpose. Please refer to the INTRODUCTION section on how to execute the WINDOWS version of the program.

If you like to generate graphical plots in WINDOWS environment, please do the following. First, activate the ASCII DATA OUTPUT window. Pull down the EDIT menu, click SELECT ALL, and click COPY. This will copy the entire file content to the CLIPBOARD. Now you can activate the CLIPBOARD and save its contents to a file, or paste it into any WINDOWS applications, such as QUATTRO, SIGMA PLOT, EXCEL, STANFORD GRAPHICS, etc.

14. ICX, ICX1, ICY:

The built-in graphics subroutine can generate two dimensional plots of any pair of parameters specified by ICX and ICY. You can generate as many plots as you need. Furthermore, the user can specify what the plot covers, i.e., plot for embedded portion of the mooring line only (ICX1 = 1), plot for catenary portion of the mooring line only (ICX1 = 2), or plot for the entire mooring line (ICX1 = 3). The plots on the screen will show the lower and upper limits of two parameters in absolute and relative scales with appropriate headings.

15. P-DELTA relation:

The program can be used to estimate the horizontal movements of the surface buoy (DELTA) at various horizontal loads applied at the buoy(P). The load varies from zero to maximum, i.e., the deployment load. The details of the P-DELTA relation are described in Section V.

SECTION III - OUTPUT

The first part of the output displays the title of the output and the file names, including the output file, the ASCII data file, and the duplicate input data file. Next, the input data are echoed, including the solution and error criteria, the mooring line properties, the geometric and material properties, the soil shear strength properties, the initial angle of the mooring line at the seafloor surface, and the sinker properties

The next part of the output contains the solution summary. This summarizes the results of the analysis showing the lengths of buried and catenary portions of the mooring line, the total length of the mooring line, the horizontal distance between the anchor and buoy, the developed tensile forces at the anchor and seafloor surface, the mooring line angles at the anchor, seafloor surface and buoy, and the details of the mooring line used in the final mooring line configuration at the time of deployment.

Following the solution summary, complete results of the analysis are printed. Included are:

- (1) Horizontal distance from the anchor measured positive toward the buoy,
- (2) Vertical distance from the seafloor surface measured positive toward the water surface,
- (3) Length of mooring line from the anchor,
- (4) Mooring line axial tensile force, and
- (5) Mooring line angle to the horizontal.

These five parameters are calculated at the end of each element. This is followed by the total elastic stretch of the mooring line at the time of deployment.

The next section of the output describes the P-DELTA relationship at the buoy. The number of P-DELTA calculation points have been selected so that all transition points where sudden changes in behavior may occur are considered. Those points include every segment ends where sinkers are attached and/or where the mooring line changes its material or diameter. A minimum of 11 points are used to describe the P-DELTA relationship for single segment mooring lines with no sinker attached.

The results are written in a tabular form on the output file, which can be viewed with any editor program, e.g., MORE command in DOS, EDIT program included in DOS, or NOTEPAD program included in WINDOWS. The results include

- (1) horizontal load at the buoy
- (2) horizontal movement of the buoy
- (3) length of the mooring line lying on the seafloor surface
- (4) beginning angle of the suspended catenary from the seafloor surface into the water
- (5) length of the suspended catenary within the water
- (6) elastic stretch of the suspended catenary within the water
- (7) elastic stretch of the mooring line lying on the seafloor surface
- (8) elastic stretch of the buried portion of the mooring line due to its axial stiffness.

Section - IV shows several example problems with detailed output results.

SECTION - IV EXAMPLE PROBLEMS

Three example problems have been selected for detailed analysis to illustrate the use of the program. They are

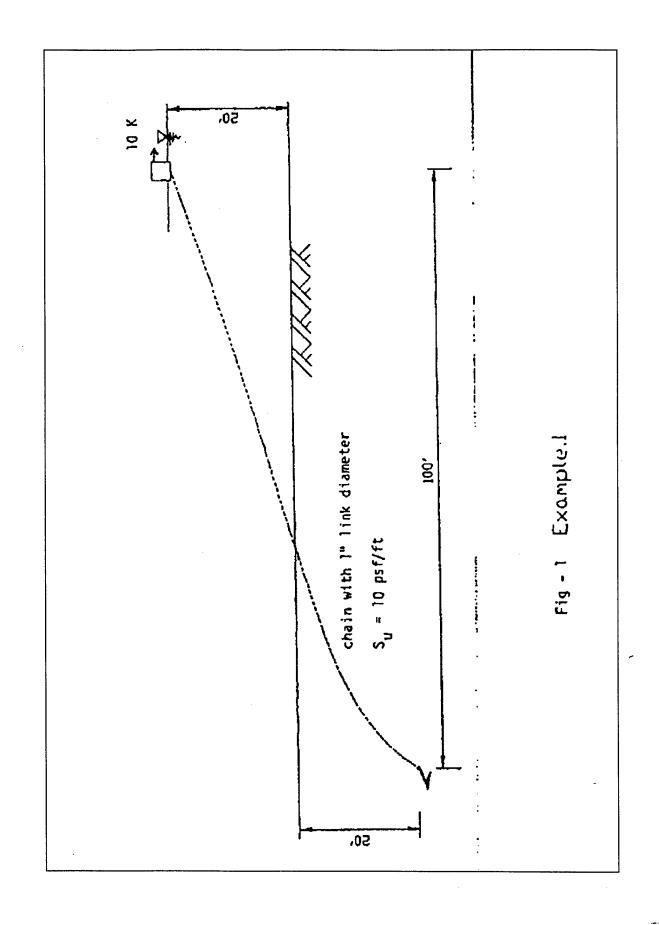
Example 1: A single segment mooring line with no sinker.

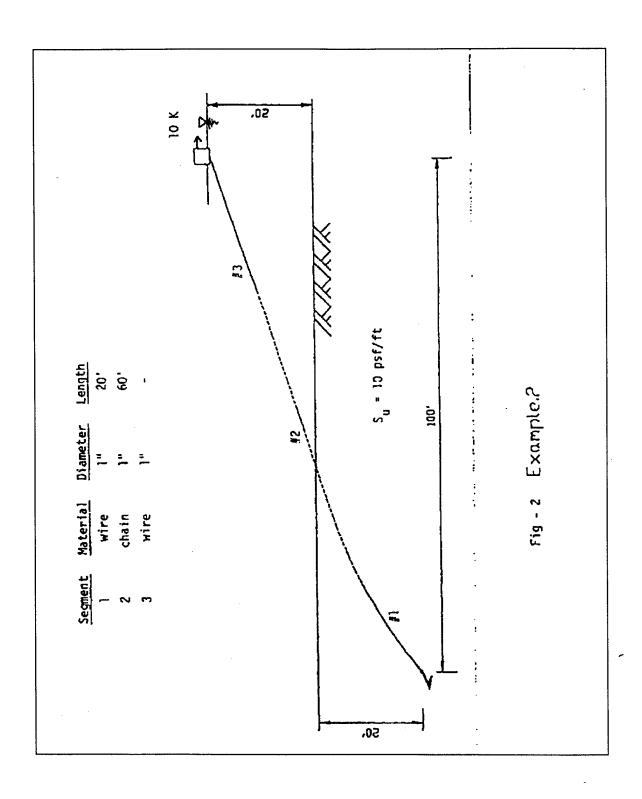
Example 2: A multi-segment mooring line with no sinker.

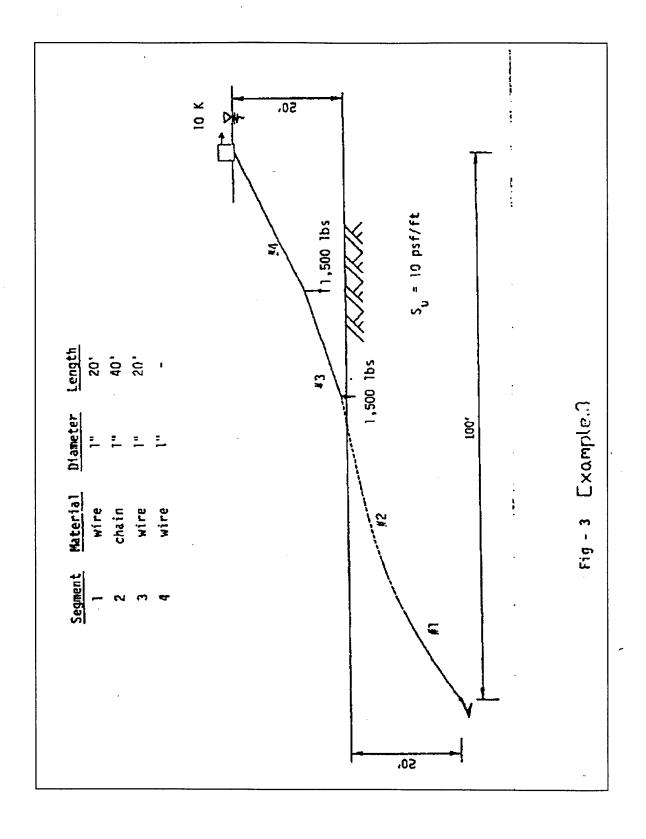
Example 3: A multi-segment mooring line with two sinkers.

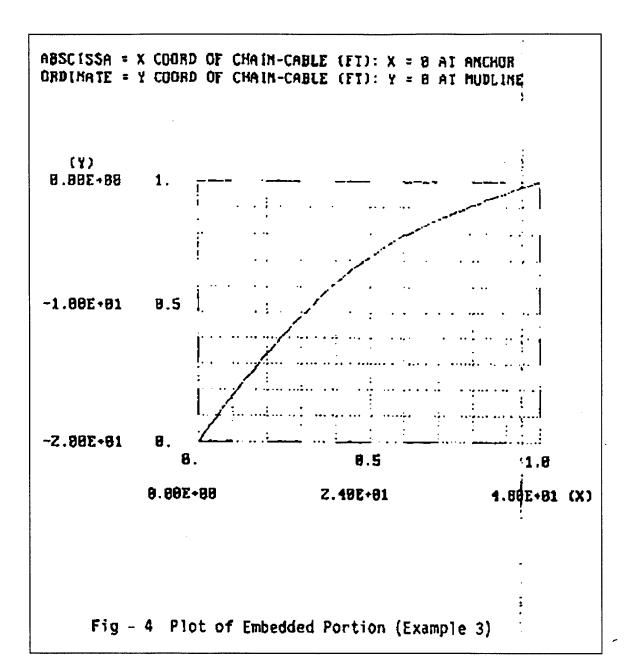
The schematics of these examples are shown in Figs - 1, 2, and 3. Included in the figures are the material, geometric, and loading properties that define the problems.

Referring to Sections I, II, and III, Tables - 1, 2, and 3 for output are provided. Figs - 4, 5, and 6 show the plots generated by the program for Example 3, displaying the embedded portion, catenary portion, and complete mooring line configurations, respectively.









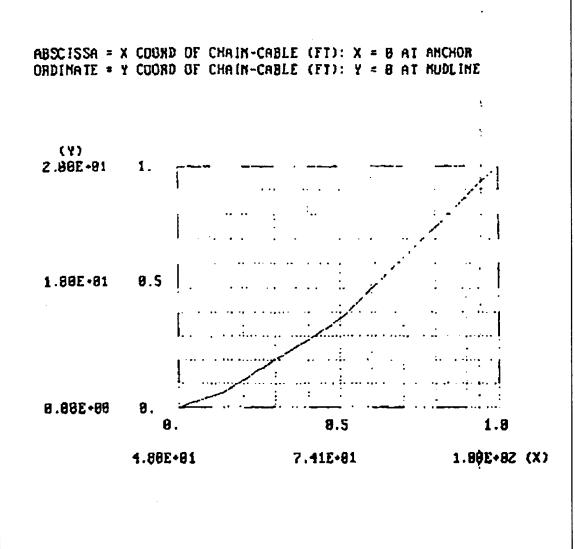
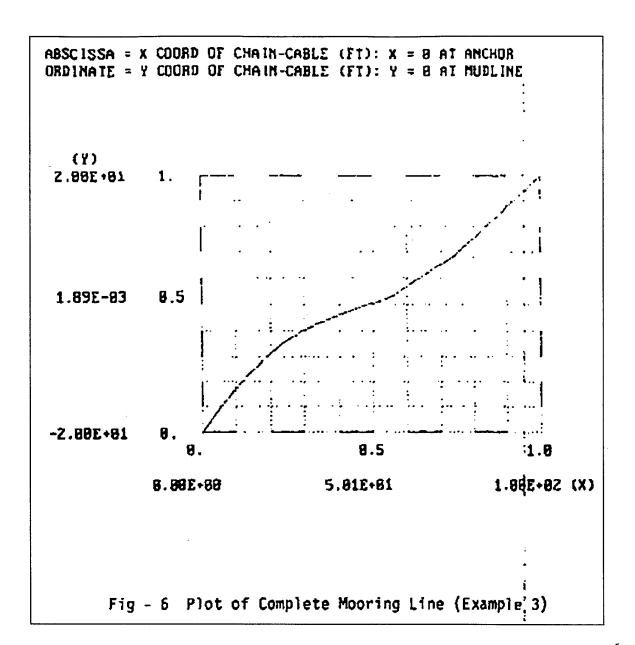


Fig - 5 Plot of Catenary Portion (Example 3)



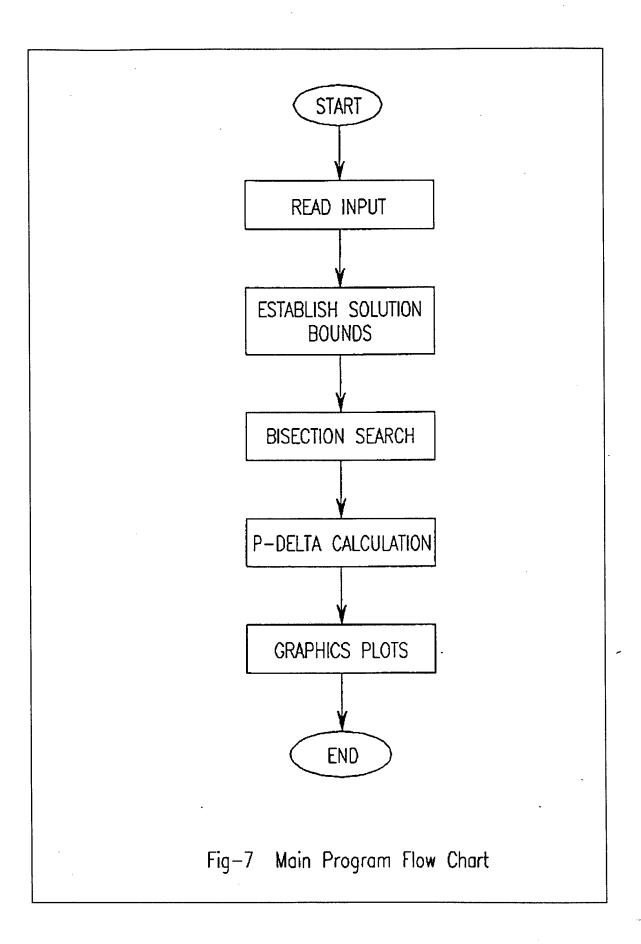
SECTION - V PROGRAM DETAILS

Complete flow charts detailing the steps involved in the analysis of mooring lines at the time of deployment and in the calculation of the P-DELTA relationship at the buoy are shown in Fig - 7 through Fig - 17. Following is a brief description of the steps involved.

- 1) Input necessary geometric, material and loading properties for the solution at the time of deployment.
- 2) Establish the solution upper and lower bounds of the mooring line angle at the seafloor surface. This is not needed if the angle at the seafloor is specified as the solution criterion.
- 3) Start the iteration using the bisection search method.
- 4) When the solution is completed, plots are displayed, if applicable.
- 5) Calculate the P-DELTA relationship at the buoy.

Note that in case any mismatch occurs among the inputted parameters the program stops the computation and asks the user to reenter a new set of input parameters. Several possible reasons for a mismatch include: (1) the mooring line size is too large or too small for the given force at the buoy, (2) the anchor is located too deep, (3) the specified total length of the mooring line or the distance between the anchor and the buoy are too long or too short, or (4) the initial angle of mooring line to the horizontal at the seafloor surface is too high.

In step 2), the mooring line exit angle at the seafloor surface starts with the inputted value or with the default value (0.1 degrees to avoid any numerical problem) and increases by 5 degrees until the inputted solution criterion (XG) lies between the calculated values from two consecutive mooring line exit angles that are 5 degrees apart.



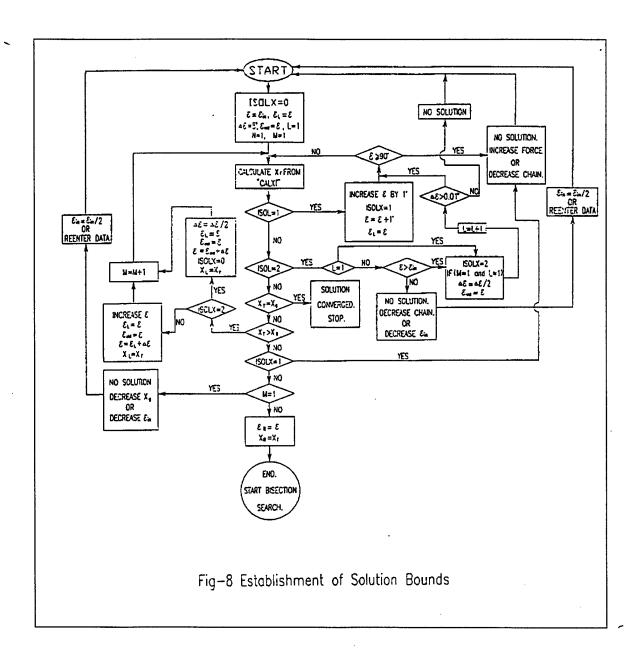
These two exit angles become the upper and lower bounds for the bisection solution search in step 3). The bisection search continues until the absolute relative error is less than ERMAX as specified by the user, i.e.,

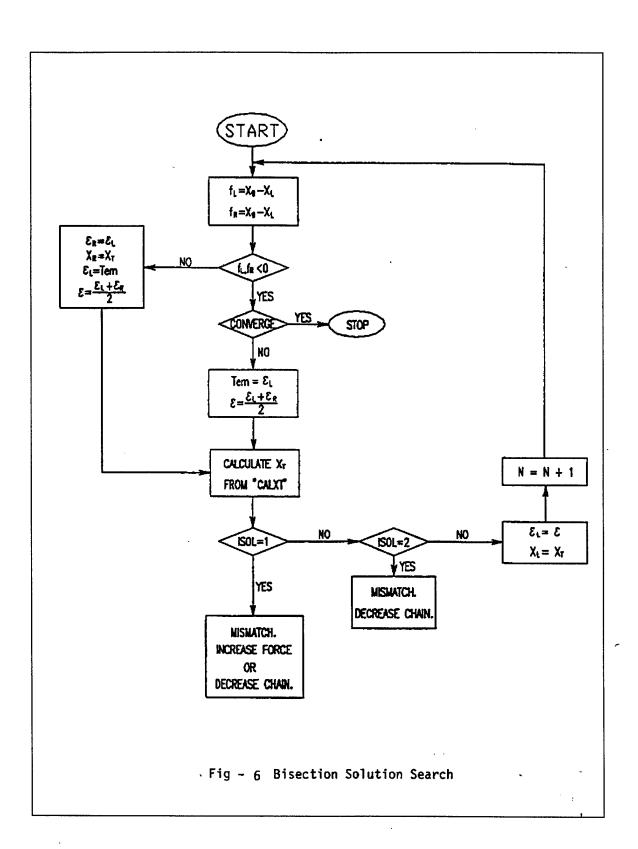
$$\frac{X \text{calculated} - XG}{XG} \le ERMAX$$

However, during the search of solution bounds, the exit angle increases by 1 degree, rather than 5 degrees, if the mooring line axial force becomes zero before the specified anchor depth is reached. On the other hand, if the mooring line angle becomes greater than 90 degrees, the step size between the previous and current iterative values of the exit angle is reduced by one half and the search continues. These are illustrated in Fig - 8. Additional factors that contribute to the decision making process include the mooring line size and the magnitude of the force applied at the buoy.

Once the upper and lower bounds of the mooring line angle at the seafloor surface are obtained, the analysis method searches for the solution iteratively, based on the method of bisection (step 3 and Fig - 9). The detailed solution is obtained from the subroutine CALXT (Fig - 10). Within the iterative solution search, the tension and angle of the mooring line are continuously monitored to identify whether any mismatch among the input parameters occurs or not, e.g., the adequacy of the specified mooring line size and the applied force at the buoy against the material and geometric properties.

The solution method however requires a known length of the buried portion of the mooring line to correctly identify the mooring line segments. A straight line is assumed between the anchor and the buoy for this purpose when either the total length of the mooring line or the horizontal distance between the anchor and buoy is specified (IS = 1)





FIGURES 8 AND 9 LEGEND

 ε : mooring line exit angle at the seafloor surface

 $\varepsilon_{\rm L}$: lower bound of ε

 ε_R : upper bound of ε

 ε_{old} : value of ε at previous iteration

X_g: solution criterion (see steps 5 and 9 in Section II or III)

 X_T : calculated value that will be compared with X_g .

 X_L : lower bound of X_T

 X_R : upper bound of X_T

N, M: counters

ISOL: 1 if axial force becomes negative

2 if mooring line angle becomes greater than 90 degrees

ISOLX: value of ISOL at the previous iteration

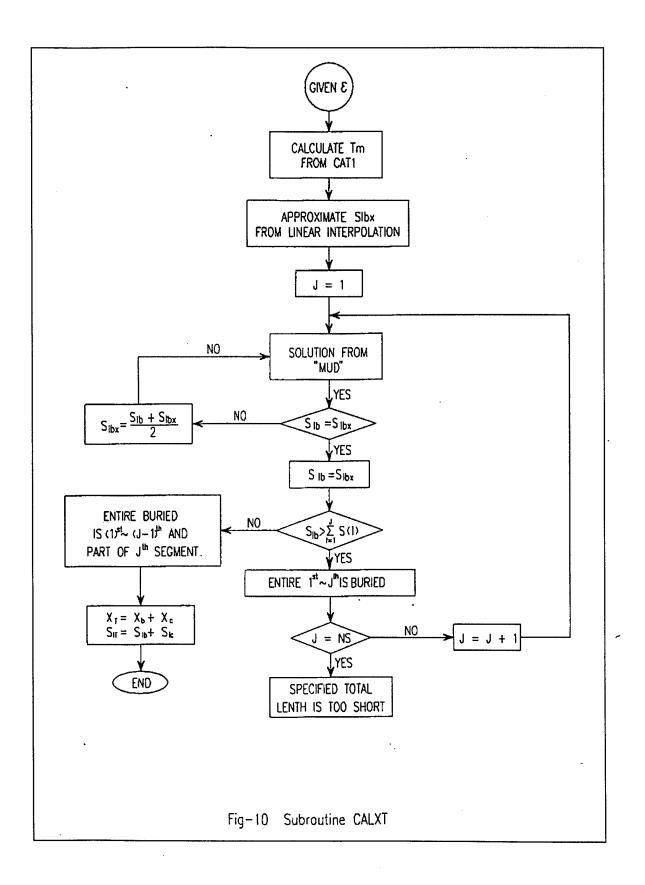


FIGURE - 10 LEGEND

X_c: horizontal length of the catenary portion of the mooring line

X_b: horizontal length of the buried portion of the mooring line

X_T: total horizontal length of the mooring line

S_{1c}: total length of the catenary portion of the mooring line

S_{lb}: total length of the buried portion of the mooring line

S_{1t}: total length of the mooring line

S_{lbx}: previous value of S_{lb}

S(I): length of the Ith segment mooring line

T_m: axial force at the seafloor surface

NS: total number of mooring line segments

J: mooring line segment counter

or 2). If the angle at the seafloor surface is specified (IS = 3), it is assumed that the buried length of the mooring line is a circular arc with the angle at the anchor being 90 degrees minus the anchor depth in feet to the vertical. When the anchor depth is greater than 90 ft., the mooring line at the anchor is set to be vertical.

The results of the analysis with these assumed parameters are then compared with the true material properties, which in turn determines the necessity of additional iterations. Iteration stops when the relative error between two consecutive solutions is less than the value specified by the user.

The detailed solution of a multi-segmented mooring line embedded in a layered seafloor soil is obtained from an additional iterative search. The initial material and geometric parameters to be used in the analysis are obtained from the assumed length of buried mooring line as described previously. The results of the analysis with these assumed parameters are then compared with the true material properties of the multi-segmented mooring line, which in turn determines the necessity of additional iterations. For instance, at the first iteration, by comparing the solution of the buried length of the mooring line with the specified length of the first segment of the mooring line attached to the anchor, one can determine whether the entire first segment is completely buried or not. If it is determined that only part of the first segment of the mooring line is buried within the seafloor, complete solutions of the buried portion and the catenary portion of the mooring line are obtained from an additional iterative search. On the other hand, if it is determined that the entire first segment of the mooring line is buried, the buried portion of the mooring line is analyzed assuming that the remaining portion of the buried length of the mooring line beyond the first segment consists of second segment mooring line.

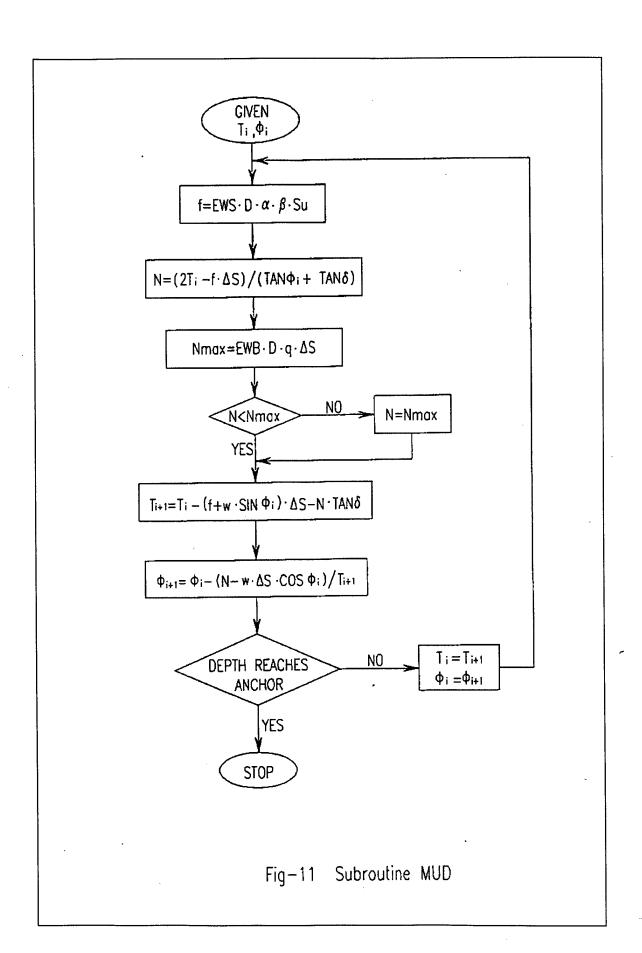


FIGURE - 11 LEGEND

T_i: axial force at the beginning of the element

 T_{i+1} : axial force at the end of the element

 ϕ_{i} : orientation angle at the beginning of the element

 ϕ_{i+1} : orientation angle at the end of the element

N: normal force

f: tangential stress

w: buoyant weight per unit length of the element

 δ : interface friction angle

q: soil bearing capacity

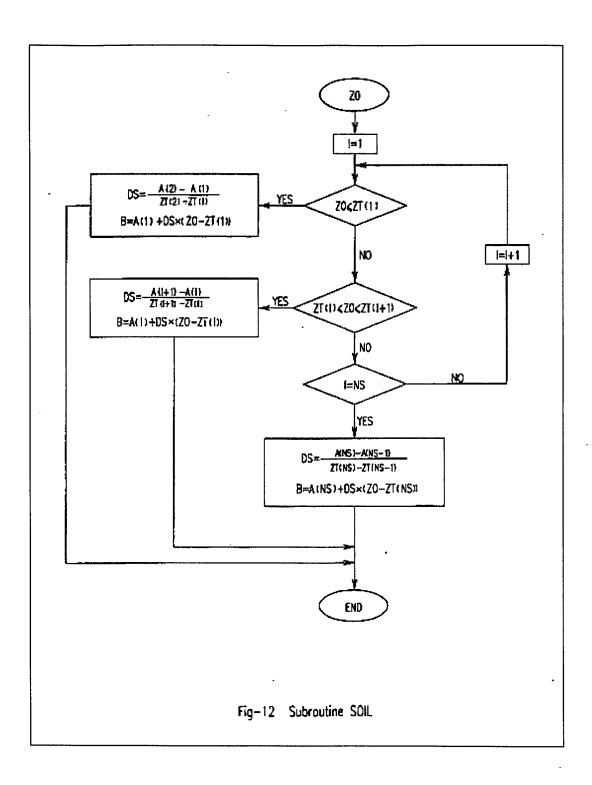


FIGURE - 12 LEGEND

ZO: depth where soil properties are desired

ZT(I): depth where soil properties are specified

NS: number of soil properties data points

A: inputted soil property at ZO

B: interpolated or extrapolated soil property

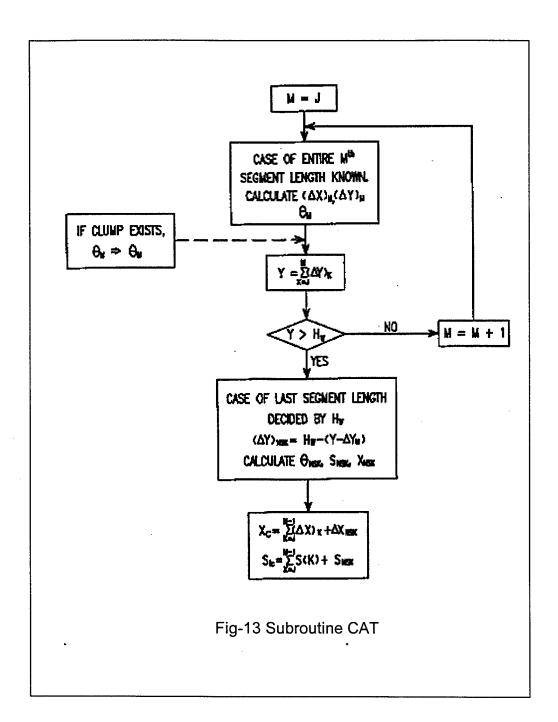


FIGURE - 13 LEGEND

 $(\Delta X)_M$: horizontal length of the Mth segment

 $(\Delta Y)_M$: vertical length of the M^{th} segment

 θ_{M} : angle of the mooring line at the end of M^{th} segment

H_w: water depth

X_c: horizontal length of the catenary portion of the mooring line

S_{lc}: total length of the catenary portion of the mooring line

NSK: total number of mooring line segments

X_{NSK}: horizontal length of the last mooring line segment

S_{NSK}: length of the last mooring line segment

 θ_{NSK} : angle of the mooring line at the water surface

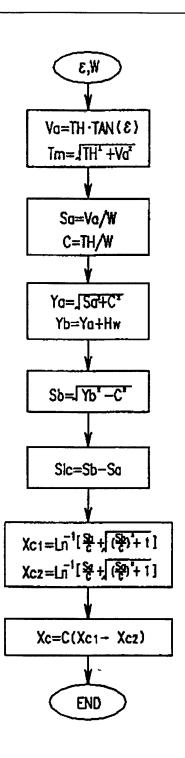


Fig-14 Subroutine CAT1

FIG - 14 LEGEND

X_c: horizontal catenary length

TH: horizontal force applied

H_w: water depth

 S_{lc} : total length of the catenary

arepsilon : catenary angle at the seafloor surface

W: buoyant weight per unit length

Similar analysis as described above will reveal whether the entire second segment of the mooring line is buried within the seafloor or not, which in turn determines the necessity of further iterations. This process continues until the complete details of the mooring line composition within the seafloor are identified, which is followed by the analysis of the catenary portion of the mooring line. The iteration stops when the relative error between two consecutive solutions of the specified criterion - the total length of the mooring line or the horizontal distance between the anchor and buoy - is less than the value specified.

The subroutine CALXT utilizes two additional subroutines - MUD and CAT. The subroutine MUD performs the recursion analysis of the mooring line from the seafloor surface to the anchor (Fig - 11). Please refer to Appendix - 1 for the details of the analysis. The soil material properties are estimated through interpolation and extrapolation in the subroutine SOIL (Fig - 12). The subroutine CAT performs the catenary analysis of the mooring line from the seafloor surface to the buoy (Fig - 13). The subroutine CAT allows as many sinkers (up to 10) within the water as specified. The details of the solution of the catenary portion of the mooring line is given in Appendix - 2. In case the catenary consists of a single segment mooring line, subroutine CAT1 (Fig - 14) is used instead of subroutine CAT.

When the solution converges, the DOS version of the program allows the graphical display of the results. Subroutine PLOT1 performs this. Subroutine PLOT1 allows any of the following monitor types: CGA noncolor, CGA color, OCGA, EGA mono, EGA color 64K, EGA color 256K,, OEGA color, VGA, OVGA, MCGA, or HGC. The program automatically determines the graphics mode available to the computer. If

an acceptable graphics mode is not available, a message (error: can not set graphics mode) will appear and the program terminates.

The plots include 11 x 11 grids for the solution domain. Both absolute and relative scales along the abscissa and ordinate are shown with appropriate captions. Any display on the monitor can be printed using the "PRINT SCREEN" key.

The load vs. deflection (P-DELTA) relationship at the buoy can be calculated at the end of the mooring line analysis. If this option is selected, a relationship between the various magnitudes of loads at the buoy and the corresponding horizontal displacements of the buoy are calculated. The number of P-DELTA calculation points depends on the input parameters defining the mooring line segments. If a single segment mooring line is specified, a total of 11 P-DELTA calculation points are used. In other words, the length of the mooring line that lies on the seafloor surface at any given load at the buoy is selected as an exact multiple of one-tenth of the maximum mooring line length that can lie on the seafloor surface, i.e., the catenary length at deployment minus the water depth. In case of multiple segment mooring lines, the P-DELTA calculation points include - in addition to 11 points explained above - all segment transition points where sudden change in P-DELTA behavior may occur, i.e., where sinkers are attached or segment changes occur. This selection of calculation points establishes a complete description of the P-DELTA relationship at the buoy.

The maximum length of the mooring line that can lie on the seafloor surface is the catenary length at deployment minus the water depth. In other words, it is assumed that the catenary portion of the mooring line forms an L-shape at zero load, i.e., the mooring line is perfectly vertical within the water and the remaining length of the catenary lies

straight on the seafloor surface with no overlap. The analysis starts with a known length of the mooring line lying on the seafloor surface as explained above. When a portion of the mooring line lies on the seafloor surface, it is assumed that the beginning angle of the suspended portion of the catenary into the water is zero. This assumed length of the mooring line lying on the seafloor surface gradually decreases from its maximum to zero and corresponding loads at the buoy are calculated. When the length of the mooring line lying in the seafloor surface reaches zero, then the beginning angle of the suspended catenary portion of the mooring line into the water starts to increase from zero to 90 degrees or until the load at the buoy reaches the maximum, i.e., the deployment load. If the solution is not obtained after 50 iterations at any given step, a message will be printed on the output indicating that nonconvergency has occurred and the solution proceeds to the next step.

Since the catenary equations require a known load at the buoy, an iterative approach is used to calculate the load at the buoy which corresponds to the given length of the mooring line lying on the seafloor surface (XD) and the beginning angle of the suspended catenary into the water (ANG). Fig - 15 shows the flow chart describing the steps involved in the solution search. The analysis first establishes the upper and lower bounds of the load at the buoy before a bisection search of the exact solution (Fig - 16) begins. An increment of one fifth of the maximum deployment load has been adopted for the purpose of establishing the solution bounds.

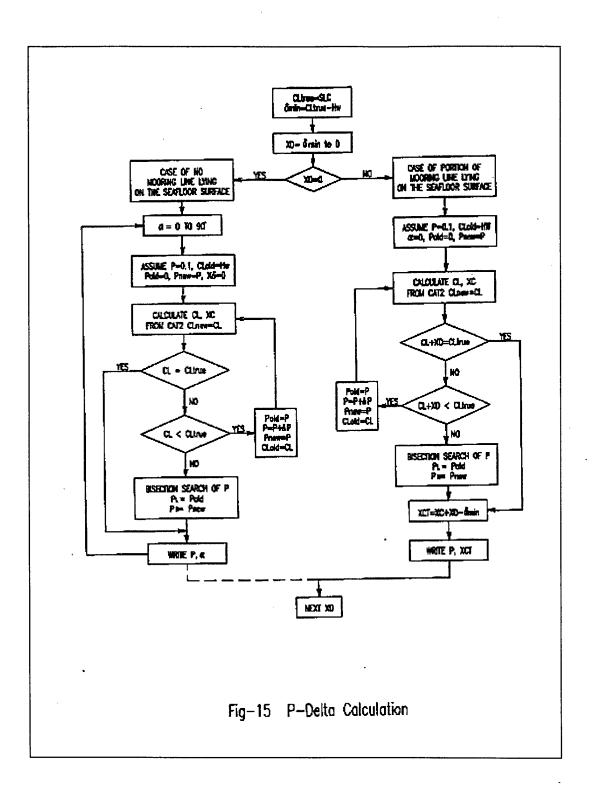
Solutions obtained from this iterative analysis, i.e., the horizontal load at the buoy (P) and the horizontal deformation of the buoy (DELTA), however, do not consider additional deformations resulting from the elastic material behaviors of the mooring line

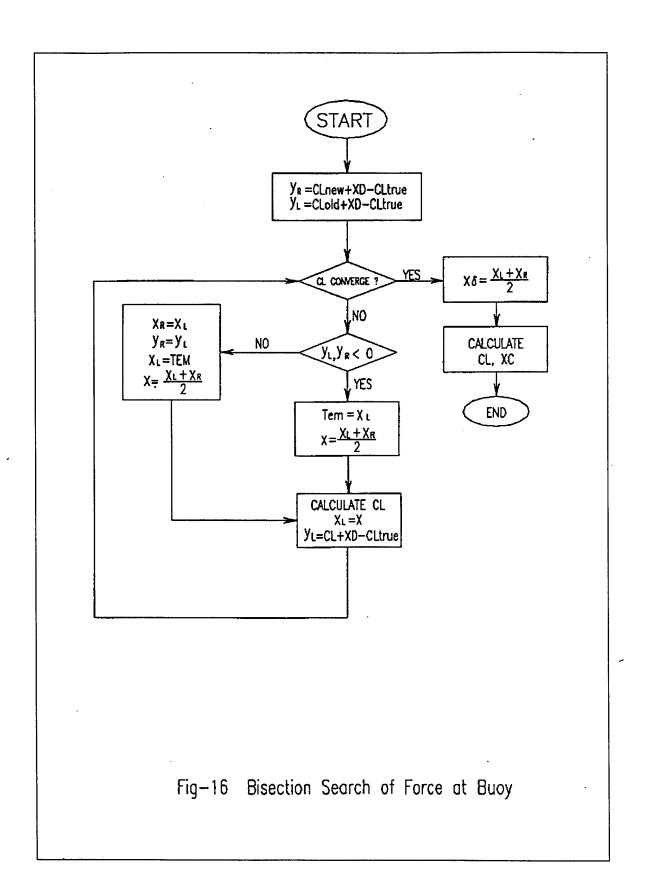
and the soil. Since a significant amount of elastic stretch may be expected, especially for relatively long mooring lines or for mooring lines made of synthetic materials, the solution needs to incorporate the large deflection effect, i.e., the increase in mooring line due to elastic stretch.

Fig - 17 shows a flow chart describing how this large deflection effect is included in the analysis. Basically, the mooring line length is continuously updated - the mooring line length is obtained from the rigid analysis and increased by the elastic stretch of the mooring line - until the difference in mooring line lengths between two consecutive iterations is less than 0.01 % in its relative error. Within each iteration, an additional iteration is performed to refine the load at the buoy, since the load changes as the mooring line length increases. The catenary length is used as a convergency check for this inner iteration loop. This inner iteration is not performed at loads of zero and deployment load, since catenary solutions do not exist when both the load and the catenary length are specified.

The elastic stretch of the mooring line consists of three major components. They are (1) the elastic stretch of the suspended portion of the catenary, (2) the elastic stretch of the mooring line lying on the seafloor surface, and (3) the stretch of the mooring line buried in the seafloor soil. They are explained separately below.

The elastic stretch of the suspended portion of the mooring line is estimated based on the applied load, the geometry of the mooring line, and the mooring line material properties. Elastic stretches of catenary elements (whose length is defined in input) within each segment (as defined by changes in material properties and geometry, and by





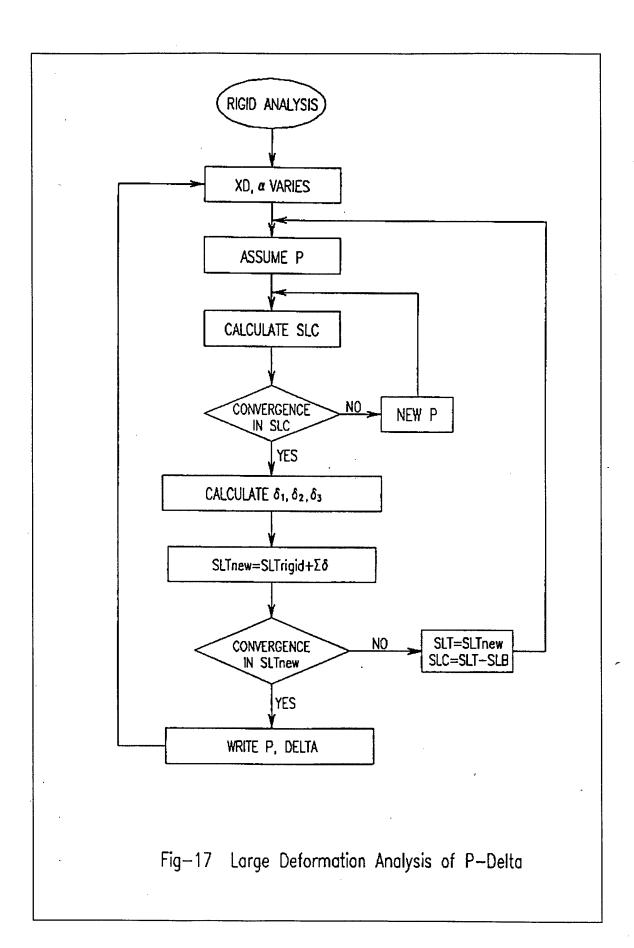


FIG - 15, 16, AND 17 LEGEND

CL: length of the suspended catenary

P: horizontal load at the buoy

SLC: length of the suspended catenary at deployment

SLT: total length of the mooring line at deployment

XC: horizontal length of the suspended catenary

XD: length of the mooring line lying on the seafloor surface

 δ_1 : elastic stretch of the suspended mooring line

 δ_2 : elastic stretch of the mooring line lying on the seafloor surface

 δ_3 : elastic stretch of the buried mooring line

 δ_{\min} : maximum value of XD

existence of sinkers) are calculated from the equation of the axial stretch of an elastic material under tension, i.e.,

$$\delta_{I} = \sum_{i=1}^{NE} \frac{P_{i}L_{i}}{(EA)_{i}}$$

where

NE = number of elements within the suspended portion of the catenary.

P_i = tension at the center of the element due to given force at the buoy.

 L_i = length of element i.

 $(EA)_i$ = axial stiffness of element i.

The elastic stretch of the portion of mooring line lying on the seafloor surface is calculated in a similar manner. However, the developed tension at the center of the element needs to be adjusted according to the tangential force that may develop between the mooring line and the soil. The developed tension at the center of the element is taken as the average of the loads at the ends of the element which differ by the tangential force occurring at the bottom of the mooring line, i.e.,

$$P_{i} = \frac{T_{L} + T_{R}}{2}$$

 T_R = force at the right end of the element

 T_L = force at the left end of the element = T_R - F

$$F = f \Delta s + W \tan \delta$$

$$f = EWS D \alpha \beta S_u$$

where F is the tangential force acting at the bottom of the mooring line. Please refer to Section II for explanations of above terms. Once the force P_i is calculated, the elastic stretch of the mooring line element can be estimated. Force at the left end of the element, T_L , then becomes the force at the right end of the next element, T_R . This process continues until the entire length of the mooring line lying on the seafloor surface is considered or the force P_i becomes zero.

The elastic stretch of the mooring line buried in the seafloor soil consists of two components; (1) the elastic stretch of the mooring line due to its axial stiffness, and (2) the elastic stretch resulting from sagging of the mooring line due to the elastic compression of the soil. To calculate these components, it is essential first to establish the load transfer mechanism, through adhesion and friction, between the mooring line and the seafloor soil. Formulation developed for the analysis of rigid mooring lines as outlined in Appendix - 1 can be applied for this purpose. It is noted that when the magnitude of the load applied at the buoy is relatively small, the tensile force developed within the mooring line may not propagate all the way to the anchor, since the soil adhesion and friction gradually decreases the mooring line tensile force as the contact area between the mooring line and the soil increases.

The elastic stretch of the buried portion of the mooring line follows the same approach as for that of the catenary. The tensile forces are calculated at the middle of each element, and resulting axial stretches are obtained and added until either the entire mooring line is considered or the tensile force becomes zero.

The stretch of the mooring line due to soil compression was not considered in this study. It is expected to be included in the future study.

APPENDIX - 1: ANALYSIS OF EMBEDDED MOORING LINE

Fig - 18 shows a schematic diagram of a mooring line element embedded in seafloor. T and ϕ are the axial tensile force and the inclination angle at the beginning of the element. N, (f ds + N tan δ), and (w ds) are the normal force, the tangential force, and the buoyant weight of the mooring line, respectively. From the equilibrium conditions of

$$\sum F_t = 0 \qquad \sum M = 0 \qquad (1)$$

one can solve for N, T and ϕ . This forms the basis of the recursion formula for the analysis of an embedded mooring line in seafloor, i.e.,

$$N = \frac{2 \operatorname{T}_{1} - f \operatorname{ds}}{\tan \phi_{1} + \tan \delta}$$

$$T_{2} = T_{1} - (f + w \sin \phi_{1}) \operatorname{ds} - N \tan \delta$$

$$\phi_{2} = \phi_{1} + \frac{N - w \operatorname{ds} \cos \phi_{1}}{T_{2}}$$
(2)

where T_1 and T_2 : axial forces at the beginning and end of the element

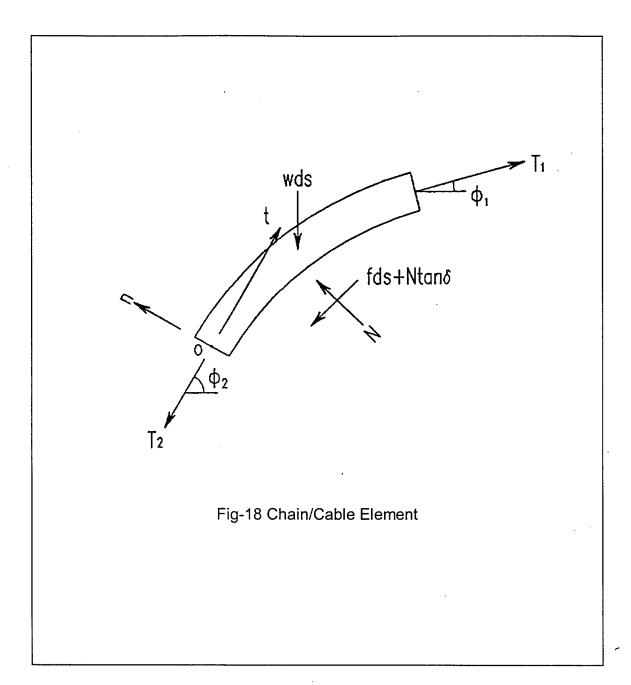
 ϕ_1 and ϕ_2 : mooring line inclination angle at the beginning and end of the element

 δ : interface friction angle between the mooring line and the soil

f: tangential force per unit length

w: buoyant weight of mooring line per unit length

N: normal force



At the seafloor surface, the inclination angle (ϕ_1) is given as part of the iteration process. From this known inclination angle and the known horizontal force at the water surface, the axial tension at the seafloor surface (T_1) is calculated. Using Eq (2), the axial tension and the inclination angle at the end of the element, ϕ_2 and T_2 , are calculated. From compatibility, the values of the axial tension and the inclination angle at the end of the current element then become those at the beginning of the next element. Concurrently, the coordinates of the elements are calculated and recorded. This process continues until the depth to the element end reaches the specified anchor depth.

The tangential component, f, is estimated from

$$f = EWS D \alpha \beta S_u$$
 (3)

where EWS: equivalent diameter conversion factor for sliding force to convert mooring line diameter to circumferential area

D: diameter of mooring line

 α : soil adhesion conversion factor

 β : contact area conversion factor

 S_n : undrained strength of soil

For chains, EWS becomes 0.94 when the chain link diameter (in inches) is converted to the circumferential area (ft²/ft) of a cylinder defined by a circle encompassing two perpendicular chain links. For cables, EWS is 0.2618. Please see Section II for further details.

The soil adhesion conversion factor, α , converts the soil cohesion into adhesion between the mooring line and the soil. The contact area conversion factor, β , is the ratio between the true contact area between the mooring line and the soil. The value of β may

be less than 1.0, if separation occurs on the back side of the mooring line while they are placed.

The interface friction angle between the mooring line and the seafloor soil, δ , is expressed as a fraction of the soil internal friction angle, ϕ . Because of the irregular shape of the outside surface of typical mooring lines, the default value of the interface friction angle is set to be 1.0 for all types of mooring line.

The value of the normal force, N, is limited to the soil bearing capacity, i.e.,

$$N < N_{\text{max}} = Q \text{ ds}$$

$$Q = EWB D q$$
(4)

where Q: bearing capacity of soil per unit length

q: bearing capacity of soil per unit area

EWB: equivalent diameter conversion factor for normal force

D: diameter of mooring line

For chains, EWB is estimated to be 0.3, while 0.0833 is used for cables. Please see Section II for further details.

The bearing capacity of the mooring line is estimated from the lateral resistance of a pile segment located at relatively deep depths. For cohesionless soils, Reese (Ref. 2) proposed the following bearing capacity equation, assuming that soil flow occurs around the pile cross-section.

$$q = K_a b \gamma z (\tan^8 \beta - 1) + K_o b \gamma z \tan^4 \beta \tan \phi$$
 (5)

where b: width of the pile

z: depth

 γ : soil unit weight

K_o: at-rest lateral earth pressure coefficient

K_a: active lateral earth pressure coefficient

 β : $45^{\circ} + \frac{\phi}{2}$

 ϕ : soil friction angle

For cohesive soils, the bearing capacity of a pile segment located at deep depths is expressed as (Ref. 3)

$$q = N_c c b (6)$$

where N_c : soil bearing capacity factor

c: soil cohesion

b: pile width

Finally the bearing capacity of the general soil having both the cohesion and the friction is calculated from the sum of equations (5) and (6), i.e.,

$$q = b \left[K_a \gamma z \left(\tan^8 \beta - 1 \right) + K_o \gamma z \tan^4 \beta \tan \phi + c N_c \right]$$
 (7)

APPENDIX - 2: CATENARY EQUATIONS

Fig - 19 shows a section of catenary. In the case when the entire segment length of the catenary is known, one can calculate the horizontal length (ΔX_c), the vertical length (ΔY_c), and the angle at the end (θ_1) from the following equations.

$$\theta_{1} = \tan^{-1}(\tan \theta_{0} + \frac{S_{c} W_{c}}{TH})$$

$$\Delta Y_{c} = \frac{TH}{W_{c}}(\sec \theta_{1} - \sec \theta_{0})$$

$$\Delta X_{c} = \frac{TH}{W_{c}} \ln(\frac{\tan(45^{0} + \frac{\theta_{1}}{2})}{\tan(45^{0} + \frac{\theta_{0}}{2})})$$

where θ_0 : angle at the beginning of segment

S_c: segment length

 W_{c} : buoyant weight of catenary segment

TH: horizontal force applied.

If a sinker is attached at the end of the segment, one can modify the value of θ_1 to

$$\theta'_1 = \tan^{-1}(\tan\theta_1 + \frac{W}{TH})$$

where W: buoyant weight of the sinker.

At the last segment of the catenary, the solution is dictated by the known vertical length of the catenary because of the specified depth of water. In that case the horizontal length (ΔX_{wr}), the segment length (S_{wr}), and the angle at the end (θ_b) are obtained from the following equations.

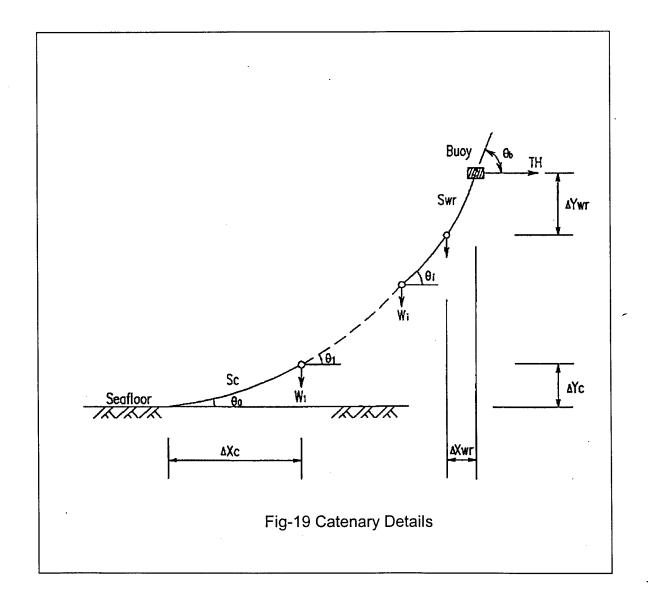
$$\theta_{b} = \sec^{-1}(\sec \theta_{1} + \frac{\Delta Y_{wr} \ W_{wr}}{TH})$$

$$S_{wr} = \frac{TH}{W_{wr}}(\tan \theta_{b} - \tan \theta_{1})$$

$$\Delta X_{wr} = \frac{TH}{W_{wr}}\ln(\frac{\tan(45^{0} + \frac{\theta_{b}}{2})}{\tan(45^{0} + \frac{\theta_{1}}{2})})$$

where ΔY_{wr} : vertical length of the catenary

 W_{wr} : buoyant weight of the catenary segment



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